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National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No. NAS-5-3760

ST - SP - RWP - 10236

NASA TT F-8987

RADAR LOCATION OF THE SUN

GPO PRICE $ ______

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Microfiche (MF) $0.50

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[USSR]

9 NOVEMBER 1964

NASA TT F-8987
ST - SP - RWP - 10236

RADAR LOCATION OF THE SUN

Geomagnetizm i Aeronomiya
Tom 4, No. 5, 825- 831,
Izdatel'stvo "NAUKA", 1964

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SUMMARY

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The expression for the energy received from a signal reflected from the corona is derived on the basis of the Gaussian distribution of the probability of radiowave scattering on the inhomogeneities of electron density. The scattering at the point of integral inner reflection is not considered. The possibility of determining the statistical distribution function of inhomogeneities is investigated, taking into account that the signal, reflected from the corona, ~~has~~ a Doppler contour.

Author

* * *

One of the important factors determining the intensity of the reflected signal at location of the corona is the characteristic of distribution of electron density inhomogeneities as a function of the radius.

Let us examine the character of radiowave propagation in the corona at location from the ground (Fig. 1). The possibility of geometric optics' approximation is substantiated in [1]. For the aiming distance we take the quantity $a = \rho d\theta$, where ρ is the distance from the antenna to the point of integral inner reflection in the corona;

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* O RADIOLKATSII SOLNTSA

θ is the angle between the radioray and the central axis.

The total refraction of the radioray is determined by formula

$$R(a) = -2\int_{1}^{n(r_0)} \frac{dn}{n} \frac{a}{[(nr)^2 - a^2]^{1/2}}, \qquad (1)$$

(see ref. [2]), where $n(r_0)$ is the value of the index of refraction at the point of total inner refraction; r is the distance from the center of the Sun to the trajectory element. The absorption is taken into account by the formula

$$\tau = \int \varkappa \, dS$$

where $\varkappa$ is the absorption coefficient.

Since in polar coordinates $dS = \sqrt{dr^2 + r^2 d\bar{\theta}^2}$ and, besides,

$$d\bar{\theta} = \frac{dr}{r} \frac{a}{(n^2 - a^2)^{1/2}} = \frac{dr}{r} \operatorname{tg} i, \qquad (2)$$

where i is the angle between the tangent to the ray and the normal to the surface of equal values of n, $\bar{\theta}$ is the angle between the central axis and the vector $\mathbf{r}$, we obtain

$$dS = dr[1 - (a^2 / n^2 r^2)]^{-1/2}. \qquad (3)$$

Let us investigate the intensity distribution of the reflected signal on the inner surface of the sphere, conducted through a transmitting antenna and having a radius of the order of magnitude of (Fig. 1).

We estimate in the first approximation that the corona has a spherically symmetric structure without inhomogeneities.

Let us fix the corona element in the form of a spherical belt of surface

$$dS_1 = 2\pi a da. \qquad (4)$$

This element is projected on the inner surface of the sphere in the form of the belt dS_2. Let us determine the area of this projection. From the triangle NPQ we have

$$P'Q = h_1 = a + \rho \sin[\pi - R(a) - d\theta] \approx a - \rho \sin R(a) + a \cos R(a), \qquad (5)$$
$$PN = h_2 = \rho \cos[\pi - R(a) - d\theta] \approx -\rho \cos R(a) + a \sin R(a).$$

Differentiating h_1 and h_2 over a from the triangle LQS, we find (QL = dH)

$$dH = (dh_1^2 + dh_2^2)^{1/2} = [(\rho R' \cos R + 2(\cos {}^1/_2 R)^2 - aR' \sin R)^2 + \\ + (\rho R' \sin R + \sin R + aR' \cos R)^2]^{1/2} da.$$

Here R = R (a), R' = dR(a)/da. Then the area $dS_2 = 2\ h_1\, dH$ is

$$dS_2 \approx 2\pi (a + \rho \sin R + a \cos R) \{[(\rho \cos R - a \sin R)R' + \\ + 2(\cos {}^1/_2 R)^2]^2 + [(\rho \sin R + a \cos R)R' + \sin R]^2\}^{1/2} da = 2\pi z(a)da. \tag{6}$$

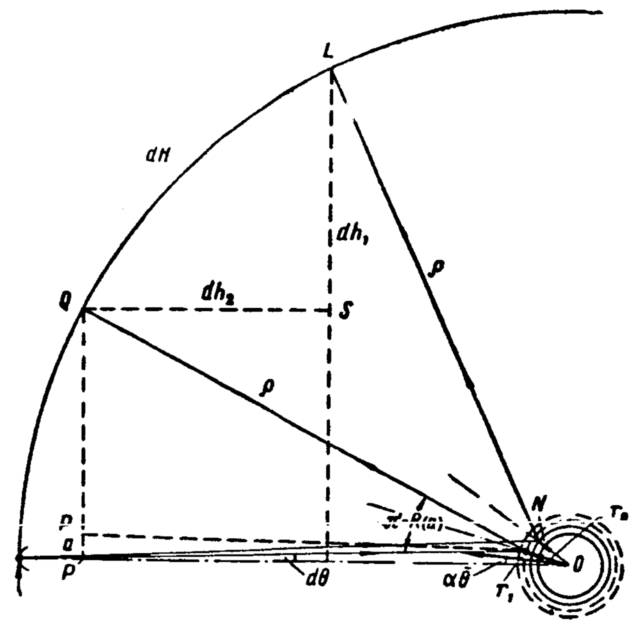

Fig. 1

The reflection of the object is characterized by the effective cross section σ, and, at the same time, we have in the geometric approximation [3]

$$\sigma = \lim_{l \to \infty} 4\pi l^2 (P_r / P_i), \tag{7}$$

where P_r is the density of the scattered energy flux of radiowaves at the distance l from the scatterer; P_i is the density of energy flux of the wave incident in the region of the object. In the case considered (see Fig. 1)

$$\frac{P_r}{P_i} = \frac{dS_1}{dS_2}\sin\frac{R(a)}{2}, \quad \sigma = 4\pi\rho^3\frac{dS_1}{dS_2}\sin\frac{R(a)}{2}.$$

Substituting the values of dS_1 and dS_2, we find

$$\sigma = 4\pi\rho^2\frac{a}{z(a)}\sin\frac{R(a)}{2} \tag{8}$$

The expression for the energy of the received signal (the radar formula) has the form

$$F = \frac{GAP_0\sigma}{(4\pi)^2\rho^4}, \tag{9}$$

where G is the directional factor of the transmitting antenna; A is the effective area of the receiving antenna; P_0 is the transmitter's energy.

The intensity of the reflected signal in an arbitrary point of the sphere will be determined, taking into account (8) and (9), as a function of the aiming distance **a**.

$$I(a) = \frac{GP_0 a \exp[-\tau_0(a)]}{4\pi\rho^2 z(a)}\sin\frac{R(a)}{2}. \tag{10}$$

In deriving (10), we did not take into account the scattering of radioawaves over the inhomogeneities of corona's electron density. We presume the absence of sharp gardients of the refractive index in the corona, and we determine the effect of inhomogeneities on the wave front deflection, determined by the laws of geometric optics. From the calculation of Chadraseckhar [4] the root-mean-square value of the angle of scattering in a medium with statistically irregular distribution of the refractive index and a correlation function of the form $\exp(-\Delta S^2/\Delta m^2)$ is determined by the expression

$$\Phi = \psi(S)\sqrt{dS}, \quad \psi(S) = 2\sqrt[4]{\Pi}\Delta n(S)\Delta m_0(S)^{-1/2}. \tag{11}$$

$\Delta n(S)$ is the root-mean-square value of the deflection of the refractive index from the unity in a single inhomogeneity; $\Delta m_0(S)$ are the mean-square dimensions of a single inhomogeneity. The root-mean-square value of the scattering angle along the whole trajectory has the form

$$\Phi_0(a) = \left[2 \int_{r_0+\Delta r}^{r(n=1)} \psi^2(S)\, dS\right]^{1/2}. \qquad (12)$$

The integration is effected to certain small vicinity of the point r_0. Substituting the value of dS from (3), we have

$$\Phi_0(a) = \left[2 \int_{r_0+\Delta r}^{r(n=1)} \psi^2(r) \frac{dr}{(1-a^2/n^2r^2)^{1/2}}\right]^{1/2}. \qquad (13)$$

Let us consider a beam of rays near the central axis, incident upon the element of the corona $dS_1 = 2\pi a da$ (Fig. 1). Upon reflection a according to the laws of geometric optics from this spherical belt, radiorays are projected on a similar element lying on the inner surface of the sphere with an area $dS_2 = 2\pi z(a) da$. Because of scattering on the inhomogeneities, a probability, distinct from zero, of wave front rotation by an angle $\bar{R}(a) = \pi - R(a)$. Since the intensity is proportional to the probability, one may estimate the energy of the signal reflected from the element dS_1, and received by the receiving antenna. The effective cross section for this element is determined by the expression

$$\sigma = 4\pi\rho^2 \frac{dS_1}{dS_2} \sin\frac{R(a)}{2} W(a), \qquad (14)$$

where $W(a)$ is the probability of deflection by the angle $\bar{R}(a)$. Then, the density of signal intensity in the region of the disposition of the transmitting antenna at the expense of reflection from the corona element dS_1 will be determined by the expression

$$I(a) = \frac{GP_0 \exp[-\tau_0(a)]}{4\pi\rho^2 z(a)} a \sin\frac{R(a)}{2} W(a)\, 2\pi h_1 da. \qquad (15)$$

At the same time, account was taken of the equations (4), (6) and (9). Accordingly, the expression for the energy of reception of the signal reflected from the whole corona will take the form

$$F = \frac{GRP_0}{2\rho^2} \int_0^{A_0} \frac{a \exp[-\tau_0(a)}{z(a)} \sin(R(a)/2)\, h_1 W(a)\, da, \qquad (16)$$

where A_0 is the radius of the diameter of corona scattering, found experimentally.

We shall take as Gaussian the probability distribution of ray angle's reflection from the geometrico-optical direction because of scattering over the inhomogeneities. As is noted in [5], the application of the latter is possible at filfillment of the condition

$$\Delta m_0 \gg \lambda_0 L, \tag{17}$$

where L is the optical path of the radioray in the corona; λ_0 is the wavelength.

If we assume that coronal rays are scattering objects, we have $\Delta m_0^{1/2} \approx 10^5$ km [6], and therefore the condition (17) can be considered as satisfied. Taking into account (12), we have

$$W(a) = \frac{c}{\sqrt{2\pi}\Phi_0(a)} \operatorname{epx}\left[-\frac{\bar{R}^2(a)}{2\Phi_0^2}\right], \tag{18}$$

where $\underline{c}$ is determined from the condition of normalization; at the same time we admit that for rays with a small aiming distance $\underline{a}$ the scattering indicatrix in the plane perpendicular to the propagation direction is a circle. Then

$$c = \left[\frac{\sqrt{2\pi}}{\Phi_0(a)} \int_0^{\pi} \exp\left(-\frac{\bar{\theta}^2}{2\Phi_0^2(a)}\right) d\bar{\theta}\right]^{-1}.$$

Substituting the two last expressions into (16) and taking into account (12), we find

$$F = \frac{GAP_0}{4\pi\rho^2} \int_0^{A_0} \frac{ah_1 \exp[-\tau_0(a)]}{z(a)} \sin\frac{R(a)}{2} \left(\int_0^{\pi} \exp\frac{-\bar{\theta}^2}{4\Psi} d\bar{\theta}\right)^{-1} \exp\frac{R^2(a)}{4\Psi} da, \tag{19}$$

$$\Psi = \int_{r_0+\Delta r}^{r(n=1)} \psi^2(s)\, ds. \tag{20}$$

As a result of the experiment, the quantity F is known, and it is now required to find the function $\Psi(s)$, which involves considerable difficulties. The problem will be simplified if the energy spectrum, conditioned by the Doppler effect, is known.

Let us consider the spectral characteristic of the received signal. The expression for the electric component has the form

$$E(t) = E_0 \cos(\omega_0 t + \varphi_0) + \sum_s E_s \cos(\omega_s t + \psi_s),$$

where ω_0, φ_0 are the circular frequency and the carrier phase; ω_s, φ_s are the circular frequency and the phase of the fringe Doppler spectrum without' Sun's emission and noises; t is the time.

Let us introduce into the consideration the quantity

$$(\omega_s - \omega_0) / 2\pi = f_s = f_0 \pm 2v / \lambda_0 \pm 2v_s / \lambda_0, \qquad (21)$$

where v is the chaotic velocity along the visual ray on account of fluctuations; v_s is the component of the constant velocity along the visual ray because of Sun's rotation. We neglect the chaotic velocity and also the differential rotation by latitude.

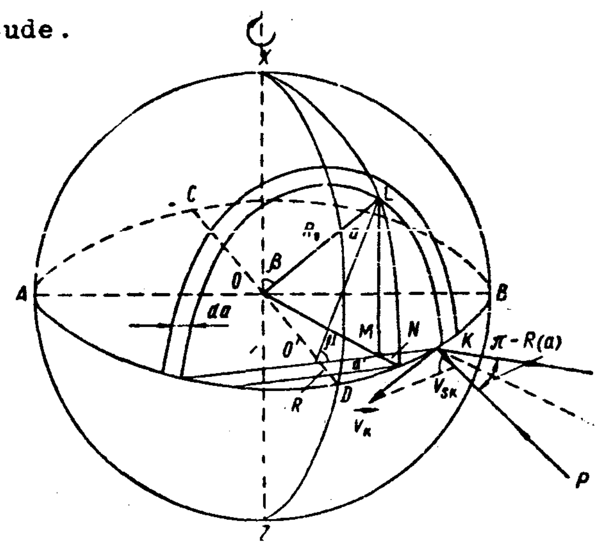

Fig. 2

Let us examine the rotation of the Sun around the axis XZ (Fig. 2). and outline an element of the corona in the form of a spherical belt. The upper part of the belt is shown in Fig. 2 . The plane ACBD is equatorial. Assume that the point of total inner reflection lies in this plane; then the incident ray PK and the reflected one KQ lie also in the equatorial plane. In reality, because of spherical stratification of the corona, the trajectory of the radioray cannot be represented by a straight line, but in the case considered, the interest is centered on the angle of refraction; it thus is appropriate to admit such a simplification. We shall take the velocity of the point K because of corona rotation equal to $\overline{V}_K$. Then, as may be seen from Fig. 2, the velocity along this point's visual ray will be

$$v_{SK} = V_K \sin {}^1\!/_2[\pi - R(a)] = V_K \cos {}^1\!/_2 R(a). \qquad (22)$$

An analogous expression will take place for the radial velocity of the point N, lying in the equatorial plane, with the difference, however, that the angle of refraction will be function of the aiming distance $\underline{a}'$, different from $\underline{a}$. Assume that the point L lies on the meridian passing through N. Since the velocity vectors of these points lie in parallel planes and coincide in direction, the radial velocity of the point L is

$$v_{SL} = V_L \cos {}^1\!/_2 R(a'). \qquad (23)$$

Let us denote $OL = ON = R_0$; $O'L = O'K = a$, $\angle LOX = \beta$, $\angle LO'M = \mu$; at the same time, because of the smallness of the corona scattering diameter relative to the total area, we may admit $R_L = \text{const}$. Then, it is easy to find from the triangles EIM, O'IM, RON

$$RN = a' = \frac{a \cos \mu}{\sin \beta}. \qquad (24)$$

From the triangles OIM and O'IM we find $\cos \beta = (a / R_0) \sin \mu$. Substituting the value found in (23), we find

$$a' = \frac{a \cos \mu R_0}{(R_0^2 - a^2 \sin^2 \mu)^{1/2}}. \qquad (25)$$

Let us determine the value of the velocity of the point L, admitting the corona rotation with an angular velocity Ω. Then $V_L = r'\Omega$. Since $r' = OM = R_0 \sin \beta$, we have

$$V_L = \Omega (R_0^2 - a^2 \sin^2 \mu)^{1/2}. \qquad (26)$$

Substituting the last expression into (24), we find the value of the radial velocity of an arbitrary point of the corona as a function of the variables a, μ

$$v_s = \Omega (R_0^2 - a^2 \sin^2 \mu)^{1/2} \cos {}^1\!/_2 [R(a, \mu)], \qquad (27)$$

where $R = R(a, \mu)$, taking into account (1) and (25), will be determined by the expression

$$R = -2R_0 \int_1^{n(r_0)} \left[(R_0^2 - a^2 \sin^2 \mu) \left(n^2 r^2 - \frac{a^2 \cos^2 \mu R_0^2}{R_0^2 - a^2 \sin^2 \mu} \right) \right]^{-1/2} \frac{a \cos \mu \, dn}{n}.$$

The expression for the frequency shift function as a consequence of the Doppler effect (21) will take the form

$$f_s = f_0 \pm \frac{2\Omega}{\lambda_0} (R_0^2 - a^2 \sin^2 \mu)^{1/2} \cos {}^1/_2 \, [R(a, \mu)]. \qquad (28)$$

Let us assign ourselves a certain interval of variables $\underline{a}$ and $\underline{\mu}$ within the limits a, $a + da$ and μ, $\mu + d\mu$; then, the differential function (28) will take the form

$$df_s = \mp \frac{2\Omega}{\lambda_0} \left\{ \left[\frac{a \sin^2 \mu \cos {}^1/_2 \, [R(a, \mu)]}{(R_0^2 - a^2 \sin^2 \mu)^{1/2}} + \frac{(R_0^2 - a^2 \sin^2 \mu)^{1/2}}{2} \sin \frac{R(a, \mu)}{2} \times \right. \right.$$
$$\left. \times \frac{dR(a, \mu)}{da} \right] da + \left[\frac{a^2 \sin^2 \mu \cos \mu}{(R_0^2 - a^2 \sin^2 \mu)^{1/2}} \cos \frac{R(a, \mu)}{2} + \right.$$
$$\left. + \frac{(R_0^2 - a^2 \sin^2 \mu)^{1/2}}{2} \sin \frac{R(a, \mu)}{2} \frac{dR(a, \mu)}{d\mu} \right] d\mu. \qquad (29)$$

To this interval da and $d\mu$ corresponds a corona area $dS_1' = a\,da\,d\mu$ and the screen area $dS_2' = z(a)\,da\,d\mu$, whence

$$\sigma(a) = 4\pi\rho^2 \frac{dS_1'}{dS_2'}, \ \sin \frac{R(a)}{2} W(a). \qquad (30)$$

Applying the theorem on the average, we find the reception energy of the signal reflected from dS'_1, taking into account (9), (12) and (18), and also the denotations (20)

$$dF(a) = \frac{GAP_0 \exp(-\tau_0(a))\, a \cdot h_1 \cdot d\mu}{8\pi^2\rho^2 z(a)} \sin \frac{R(a)}{2} \times$$
$$\times \left[\int_0^{\pi} \exp\left\{ -\frac{\bar{\theta}^2}{4\Psi} \right\} d\bar{\theta} \right]^{-1} \exp\left\{ -\frac{\bar{R}(a)^2}{4\Psi} \right\} da. \qquad (31)$$

Several such equations can be composed for various intervals da and $d\mu$. If the energy frequency spectrum of the received signal is known, the left-hand parts of these equations may be determined from the

Doppler contour upon preliminary calculation of frequency accretion from the expression (29). Thus arises the possibility of determining the distribution function of inhomogeneities. Let us represent this function in the form $\psi^2(r) = \psi_0 + \psi_1 r + \psi_2 r^2$ as was admitted in [2]. Substituting this expression into (31) and taking into account (3), (6), we find

$$dF(a) = T(a) \exp \frac{-\overline{R}^2(a)}{4\vartheta(a)} \left(\int_0^{\pi} \exp \frac{-\bar{\theta}^2}{4\theta^2} d\bar{\theta} \right)^{-1} \qquad (32)$$

Here

$$T(a) = \frac{GAP_0 \exp(-\tau_0(a)\, da \cdot a \cdot d\mu \cdot \sin {}^1/_2 R(a)}{8\pi^2 \rho^2} \times$$

$$\times \{[(\rho \cos R(a) - a \sin R(a)) R'(a) + 2(\cos {}^1/_2 R(a))^2]^2 +$$

$$+ [(\rho \sin R(a) + a \cos R(a)) R'(a) + \sin R(a)]^2\}^{-1/_2};$$

$$\vartheta(a) = \psi_0 \alpha(a) + \psi_1 \beta(a) + \psi_2 \gamma(a),$$

$$\alpha(a) = \int_{r_0+\Delta r}^{r(n=1)} \frac{dr}{(1 - a^2/n^2r^2)^{1/_2}}, \quad \beta(a) = \int_{r_0+\Delta r}^{r(n=1)} \frac{rdr}{(1 - a^2/n^2r^2)^{1/_2}},$$

$$\gamma(a) = \int_{r_0+\Delta r}^{r(n=1)} \frac{r^2dr}{(1 - a^2/n^2r^2)^{1/_2}}, \quad \overline{R}(a) = \pi + 2\int_1^{n(r_0)} \frac{dn}{n} \frac{a}{[(nr)^2 - a^2]^{1/_2}}.$$

The last quantities can be computed by the numerical method, provided the dependence n (r) is given. Let us denote the expression

$$\psi_0 \alpha(a) + \psi_1 \beta(a) + \psi_2 \gamma(a) = X(a).$$

Then the expression (32) will take the form

$$dF(a) = T(a) \exp\left\{-\frac{\overline{R}^2(a)}{4X(a)}\right\} \left(\int_0^{\pi} \exp \frac{-\bar{\theta}^2}{4X(a)} \right)^{-1} = \frac{T(a) \exp(-\overline{R}^2(a)/4X(a))}{2\sqrt{X(a)}\, erf(\pi)}. \quad (33)$$

Let us assign ouselves the discrete value of a in the interval $0 < a < A_0$. Then, upon taking the logarithm, we obtain from (33) a system of transcendential equations of the form

$$\ln \frac{T(a_1)}{dF(a_1)} = \frac{\overline{R}^2(a_1)}{4X(a_1)} + \ln \operatorname{erf}(\pi) + \ln 2\sqrt{X(a_1)},$$

$$\ln \frac{T(a_2)}{dF(a_2)} = \frac{\overline{R}^2(a_2)}{4X(a_2)} + \ln \operatorname{erf}(\pi) + \ln 2\sqrt{X(a_2)}, \qquad (34)$$

Varying a in the system (34), it is necessary to take into account that the loss of information will decrease with the decrease of intervals between the discrete a. Resolving this system relative to X (a), we shall obtain a system of the form

$$X(a_1) = \psi_0\alpha(a_1) + \psi_1\beta(a_1) + \psi_2\gamma(a_1),$$
$$X(a_2) = \psi_0\alpha(a_2) + \psi_1\beta(a_2) + \psi_2\gamma(a_2),$$
$$\cdots\cdots\cdots\cdots\cdots$$

The constants ψ_0, ψ_1, ψ_2 can be determined from the last system by the method of least squares, which will provide the possibility of estimating the statistical distribution function of inhomogeneities $\Psi(r)$.

In conclusion, I am grateful to V. M. Polyakov for his interest in this work and his remarks, and also to I. S. Shklovskiy, for presenting the theme.

*** THE END ***

IZMIRAN Received on 1 Nov. 1963.

Contract No. NAS-5-3760
Consultants & Designers, Inc.
Arlington, Virginia

Translated by ANDRE L. BRICHANT
on 8 and 9 November 1964.

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